This question paper contains 3 printed pages.

Your Roll No. .....

Sl. No. of Ques. Paper: **8509** **HC**

Unique Paper Code : **32357502**

Name of Paper : **Mathematical Modelling & Graph Theory**

Name of Course : **Mathematics : DSE for Hons.**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any three parts from each question.

SET- A

1. (a) Solve the initial value problem using Laplace transform: (6)

$$x''+6x'+25x=0;\ x(0)=2, x'(0)=3.$$

(b) (i) Find the inverse Laplace transform of (2)

$$F(s)=\frac{s-1}{(s+1)^3}.$$

(ii) Show that (2)

$$L\{t\,Cosh\,kt\}=\frac{s^2+k^2}{(s^2-k^2)^2}.$$

(iii) Find the inverse Laplace transform of (2)

$$F(s)=\frac{s^3}{(s-1)^4}.$$

(c) Find two linearly independent Frobenius series solutions of (6)

P. T. O.

$$2xy''+3y'-y=0.$$

(d) Use power series to solve the initial value problem: (6)

$$y''+xy'-2y=0;\ y(0)=1, y'(0)=0.$$

2. (a) Explain Middle-Square Method and use it to generate random numbers taking $x_0=2041$. Does this method has any drawbacks? Illustrate. (6)

(b) Using Monte Carlo Simulation, write an algorithm to calculate the volume of the sphere

$$x^2+y^2+z^2\le 1$$

that lies in the first octant, $x>0, y>0, z>0$. (6)

(c) Using graphical analysis (6)

Minimize $x-y$
subject to

$$x+y\ge 6,$$
$$2x+y\ge 9,$$
$$x, y\ge 0.$$

(d) Using simplex method (6)

Maximize $6x+4y$
subject to

$$-x+y\le 12,$$
$$x+y\le 24,$$
$$2x+5y\le 80,$$
$$x, y\ge 0.$$

3. (a) (i) Draw two non-isomorphic regular graphs with 8 vertices and 12 edges. (3)

(ii) Prove that if G is a simple graph with at least 2 vertices then G has two or more vertices of same degree. (3)

(b) (i) Determine for what values of n, r and s the graphs given below are Eulerian and Semi-Eulerian.

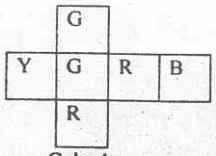
A) the complete graph K_n

B) the complete bipartite graph $K_{r,s}$

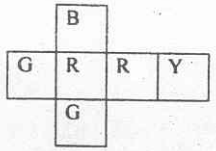
C) the $n-$cube Q_n. (4)

(ii) State Handshaking Lemma. (2)

(c) Show that there will be no solution to the four cubes problem for the following set of cubes. (6)

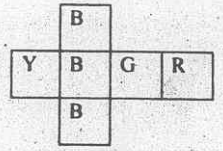


Cube 1



Cube 2

Cube 3



Cube 4

(d) Prove that there is no knight's tour on a 3 x 3 chessboard. **(6)**

4. **(a) Use the factorization:** **(7)**

$$s^4+4a^4=(s^2-2as+2a^2)(s^2+2as+2a^2)$$

and apply inverse Laplace transform to show that:

$$L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}=\frac{1}{2a^2}Sinhat\ Sinat\ .$$

(b) Solve the initial value problem: **(7)**

$$y''+(x-1)y'+y=0;\ y(1)=2, y'(1)=0.$$

(c) Fit the model $y=cx$ to the data using Chebyshev's criterion to minimize the largest deviation **(7)**

x	1	2	3
y	2	5	8

(d) Prove that if G be a graph in which every vertex has an even degree, then G can be split into cycles, such that no two cycles have an edge in common. **(7)**